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Part III. Non-Newtonian Fluids

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The use of helical ribbon agitators to mix viscous non-Newtonian fluids has been investigated. A generalized model, based on an appropriate definition of effective viscosity, is proposed to predict power consumption. This model is most successful with fluids that do not have a high degree of elasticity.

It was found that the efficiency of mixing of pseudoplastic fluids was about half of that of Newtonian fluids in the same mixer, while the efficiency of mixing viscoelastic fluids was still lower and approximately independent of the mixer geometry. Blade width was the primary variable affecting the mixing efficiency on inelastic fluids.

SCOPE

The mixing of very viscous fluids can be efficiently accomplished with a helical ribbon agitator (HRA). A model to predict the power consumption of the HRA mixing Newtonian fluids has been reported by Patterson et al. (1979); however, very viscous fluids are often non-Newtonian or viscoelastic in nature. This paper reports the experimental results of various HRA's mixing pseudoplastic and viscoelastic fluids. Six different mixer geometries with three different fluids were examined. The fluids varied from negligible elasticity (2% aqueous Natrosol) to high elasticity (1% aqueous Separan). The model previously derived has been generalized to include the mixing of non-Newtonian fluids. This was accomplished by defining an effective rate of deformation that is functionally dependent on the agitator geometry rotational speed and

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fluid properties. The functional dependence was determined for each mixer geometry by setting the generalized Reynolds number ($Re_g = d^2N\rho/\eta_e$) equal to the Reynolds number for the Newtonian fluid case. Stated differently, Re_g is defined such that the power number curves of the two fluids coincide.

The relative mixing efficiency of the various impellers is discussed for the non-Newtonian fluids. The effect of mixer geometry on the non-Newtonian fluids is compared and discussed with respect to the performance with Newtonian fluids. The problem of scale-up of helical ribbon agitators for non-Newtonian and viscoelastic fluids using the generalized model is discussed briefly.

CONCLUSIONS AND SIGNIFICANCE

A generalized model for predicting the power consumption of helical ribbon agitators has been developed. This model, based on an appropriate definition of effective viscosity, has been tested with a number of data obtained experimentally using Newtonian, non-Newtonian, and viscoelastic fluids. The standard deviation for seven widely different fluids was found to be 5%. The important geometrical variables were determined to be the vessel and agitator diameters and the blade length. The fluid parameters of importance were the zero shear viscosity, the fluid time constant, and a dimensionless parameter characterizing the non-Newtonian behavior. These rheological parameters were obtained from the Carreau (1972) viscosity model.

It was found that the mixer efficiency depended strongly on the degree of fluid elasticity, decreasing rapidly as the elasticity increased. The agitator geometry had relatively little effect on the mixing efficiency for the highly viscoelastic fluids; all agitators were about equally inefficient when compared to their behavior in Newtonian fluids. The effect of mixer geometry for the pseudoplastics follows the trend established for the Newtonian fluids. The most important variable was the blade width; the efficiency increased with the blade width. The single-blade impeller also showed a high relative efficiency, as it did for the Newtonian fluids. Generally, it is remarked that the efficiency of mixing pseudoplastic fluids was about 40 to 50% of that of the Newtonian case, while the mixing of viscoelastic was again less efficient.

The proposed use of the effective viscosity (or, equivalently, a generalized Reynolds number) allows the scale-up of mixer geometries with reasonable confidence when pseudoplastic fluids are to be mixed. However, the data show that the scale-up of mixers for viscoelastic fluids should be approached with caution. This is a limitation of the effective viscosity which does not properly account for the effects of fluid elasticity.

PROBLEM STATEMENT

In a previous paper, a model was developed for the power consumption of helical ribbon agitators (HRA's) employed in the mixing of Newtonian liquids. The concept of relative mixing efficiency was introduced and shown to be useful for comparing the performance of different HRA's. The HRA is most effective when blending very viscous fluids, where flow is restricted to the laminar regime. In practice, this means that the HRA is used to mix polymeric solutions or melts which are often non-Newtonian and can also exhibit viscoelastic behavior. In this paper, we extend the previously developed model to the mixing of non-Newtonian and viscoelastic liquids.

Although the mixing of non-Newtonian and viscoelastic liquids is an important industrial activity, very few data have been reported in the literature for the HRA. Three modifications have been proposed for the Newtonian fluid model of Bourne and Butler (1965). This model presumes that all the power consumed is dissipated in the gap between the agitator and the vessel wall. The first modification was done by Bourne and Butler (1965) who derived equations for the mixing of a power law fluid. Chavan and Ulbrecht (1972) examined an extended model, also assuming power law fluid behavior. Finally, Sawinsky et al. (1976) used an empirical modification of Bourne and Butler's model for pseudoplastic fluids that was within 20% of experimental data and about equal to the accuracy of Chavan and Ulbrecht's predictions. We have been unable to find any reported data for the mixing of viscoelastic fluids using the HRA, although the mixing of these fluids with a central screw added to the agitator has been described by Chavan et al. (1975) and Coyle et al. (1970).

THEORETICAL CONSIDERATIONS

A model for the power consumption of HRA mixing Newtonian fluids was derived by Patterson et al. (1979) considering drag forces exerted on the impeller blades. Unfortunately, this method is not feasible for non-Newtonian and viscoelastic fluids. The complex flows involved exclude this method even when using the simplest constitutive equation for viscoelastic fluids. Instead, we follow Metzner and Otto (1957) and define an effective shear rate $\dot{\gamma}_e$ proportional to the agitator rotational speed:

$$\dot{\gamma}_e = kN \quad (1)$$

This, together with a dimensional analysis and a viscosity model, provides a method for predicting the power consumption. The dimensional analysis yields for a power law fluid (Skelland, 1967)

$$N_p = \frac{P}{\rho N^3 d^5} = f\,(Re_g, Fr, n, \text{geometrical ratios}) \quad (2)$$

For geometrically similar systems, the relationship reduces to

$$N_p = f\,(Re_g, Fr, n) \quad (3)$$

It is obvious that Equation (3) cannot account for any effects owing to the elastic nature of the fluid.

Viscoelastic Fluids

The behavior of viscoelastic liquids in any flow system should depend on their elastic response. There are a large number of constitutive equations available which describe, more or less exactly, the behavior of viscoelastic fluids in different flow situations. Although, as pointed out earlier, an analytic solution is proscribed by the complex flow in a mixer, the rheological behavior can be simply expressed in terms of the parameters obtained from these equations and then used in the dimensional analysis of mixing.

Effective Rate of Deformation

The flow in a mixer is three dimensional, and the fluid is accelerated and decelerated as it passes around the blade. Following the suggestion of Metzner and Otto (1957), we assume that, on the average, the rate of deformation is a linear function of the impeller rotational speed as given by Equation (1). This expression has been tested for a large variety of sizes and types of impellers (turbines, propellers, and paddles) operating in Bingham and pseudoplastic fluids. It was implied that the characteristics of non-Newtonian fluids, the type of impeller, and its dimensions and that of the mixing vessel are without influence on k. Although this equation was later supported by Calderbank and Moo-Young (1959) and Godleski and Smith (1962), none of them discussed the actual confidence of their data. The ±30% deviation found in the data likely implies that the constant k is affected by the material properties and system geometries. The proposed function of Calderbank and Moo-Young (1959) for dilatant materials which shows a dependence of $\dot{\gamma}_e$ on the geometries is an obvious example. In this study, the effective rate of deformation is found to be a function of the fluid rheological properties, geometrical configuration of the impeller vessel, and rotational speed of the impeller.

Selection of Constitutive Equation

In previous studies, the power law model with two parameters was used to describe the rheological properties of non-Newtonian fluids. However, the power law model is rather inadequate to express the properties of non-Newtonian fluids over a wide range of shear stress or shear rate. However, it is assumed that the simple shear non-Newtonian viscosity is adequate to describe the viscous properties of the fluid around the blade. This is valid for low Weissenberg number; that is, the ratio of the characteristic elastic time of the fluid to the characteristic time of deformation rate is small. This assumption is reasonable for low elasticity fluids (most polymeric solutions do not exhibit important elastic properties) and low values of impeller rotational velocities (as normally used in helical-ribbon agitators).

TABLE 1. PROPERTIES OF FLUIDS

Fluid	ρ (kg/m³)	η_o ($N \cdot s/m^2$)	S	t_1 (s)	λ* (s)
100% glycerol	1 254	0.568	—	—	—
100% glycerol	1 249	0.800	—	—	—
Vitrea oil	869	0.193	—	—	—
1.0% Natrosol 250-HR	1 000	1.07	0.235	0.233	—
1.5% Natrosol	1 000	24.0	0.381	1.30	—
1.5% CMC-7H	1 000	2.5	0.175	0.437	0.12
2.0% CMC-7H	1 000	10.0	0.244	1.26	0.052
0.8% Separan AP-30	1 000	340	0.382	99.3	0.59
1.0% Separan AP-30	1 000	1 100	0.392	298	0.70
1.5% Separan† AP-30	1 000	1 200	0.417	145	0.60

* The characteristic elastic time constant was calculated from rheological data at a shear rate equal to 10 s⁻¹ through the relation $\lambda = \tau_{11} - \tau_{22}/\tau_{12}\,\dot{\gamma}$.

† Aged polymer powder.

The viscosity model proposed by Carreau (1972)

$$\eta = -\frac{\tau_{12}}{\dot{\gamma}} = \frac{\eta_0}{[1 + t_1^2\,\dot{\gamma}^2]^S} \quad (4)$$

is used to characterize the fluid in the mixing system. This equation has been shown by Hill (1972) to describe very well the viscosity data of a large class of polymeric fluids. For the mixing system, the shear rate is replaced by the effective rate of deformation in the form of Equation (1); then the effective viscosity is expressed as

$$\eta_e = \frac{\eta_0}{[1 + t_1^2\,\dot{\gamma}_e^2]^S} \quad (5)$$

For large enough values of $\dot{\gamma}_e$, this reduces to

$$\eta_e = \frac{\eta_0}{(t_1\,\dot{\gamma}_e)^{2S}} \quad (6)$$

which is equivalent to a power law expression. Thus, the generalized Reynolds number can be written as

$$Re_g = \frac{d^2\,N\,\rho}{\eta_e} \simeq \frac{d^2\,N\,\rho\,(t_1\,\dot{\gamma}_e)^{2S}}{\eta_0} \quad (7)$$

Froude Number

The Froude number N^2d/g represents the ratio of inertial force to gravitational force. In most cases, agitation is carried out with a free liquid surface in the tank. The shape of the surface and thus the flow patterns in the vessel are affected by the gravitational force. This is particularly noticeable in unbaffled tanks where vortexing occurs, the shape of vortex representing the balance between gravitational and inertial forces. However, in this work, within the experimental range of rotational speeds employed, no vortex was formed. Therefore, the Froude number was not a significant variable and is subsequently neglected.

EXPERIMENTAL

The experimental procedure and equipment used is that described in Patterson et al. (1979). The nomenclature for the mixer geometries and agitators employed is identical and is not repeated here. The reader is referred to Yap (1976) for more complete details. The fluids were characterized experimentally using a Weissenberg rheogoniometer, model R-18. Their properties pertaining to the Carreau model are summarized in Table 1. The aque-

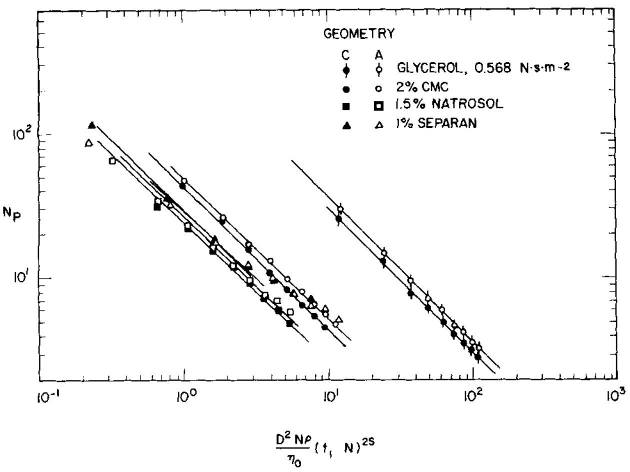

Fig. 1. Determination of the factor ϕ_2 for various fluids in geometries A and C.

TABLE 2. SHIFT FACTOR OF THE POWER CURVE ϕ_2

Geometry*	$(d/D)^2 (l/d)$	2% CMC	ϕ_2 1.5% Natrosol	1% Separan
A	3.63	7.1	11.4	10.7
B	2.43	5.7	9.6	6.1
C	3.34	6.9	11.2	10.6
D	2.13	5.2	7.1	3.8
E	3.56	6.7	10.8	3.4
F	3.60	7.4	11.5	11.2

* For a more detailed description of the geometries, refer to Patterson et al. (1979), Table I.

ous polymer solutions were freshly prepared for the tests and experiments since experience has shown that their rheological properties change if they are stored for more than a few weeks. The biodegradable fluids (CMC and Natrosol) were protected by the addition of 0.01% phenylmercuric acetate which acted as a bactericide and fungicide.

RESULTS AND DISCUSSION

For polymeric fluids, the effective rate of deformation is defined by

$$\dot{\gamma}_e \equiv \phi_1 N \tag{8}$$

where ϕ_1 is a function of the fluid rheological parameters and, possibly, of the geometrical parameters. The functional dependence of ϕ_1 is obtained from the power data by comparing the power number of a given viscoelastic fluid with that of a Newtonian fluid in the same geometry. Then, at the same power number, set $Re_g = Re$. This condition specifies that the generalized Reynolds number must be defined such that the power curves of the two fluids coincide. Equation (9) then follows from the definition of the generalized Reynolds number

$$\frac{d^2 N_1 \rho_1}{\eta_0} (t_1 \phi_1 N_1)^{2S} = \frac{d^2 N_2 \rho_2}{\mu_2} \tag{9}$$

such that $N_{P_2} = N_{P_1}$. Defining

$$\phi_2 \equiv \phi_1^{2S} \tag{10}$$

we see that ϕ_2 is the shift factor between the log-log plots of curves of N_p vs. $d^2 N\rho(t, N)^{2S}/\eta_0$ and N_p vs. $d^2 N\rho/\mu$. A Newtonian fluid has $t_1 = 0$, $S = 0$ and $\eta_0 = \mu$; thus, the generalized Reynolds number can be used for both types of fluids. Typical curves for geometries A and C are shown in Figure 1. The curves for the other four geometries are similar and are not presented here. For simplicity, the slope of the curves were taken to be -1, although the 1% Separan solution shows large deviations from that value in some geometries. The solid line for the Newtonian fluid is the prediction of Equation (24) of Patterson et al. (1979). The values of ϕ_2 obtained for the three viscoelastic fluids and six geometries are presented in Table 2. This table clearly shows the dependence of ϕ_2 on the fluid properties and on the geometrical ratios.

An attempt to correlate the shift factor ϕ_2 with the geometrical variables and fluid properties gave

$$\phi_2 = 4\left[\left(\frac{d}{D}\right)^2 \left(\frac{l}{d}\right)\right]^{2S} \tag{11}$$

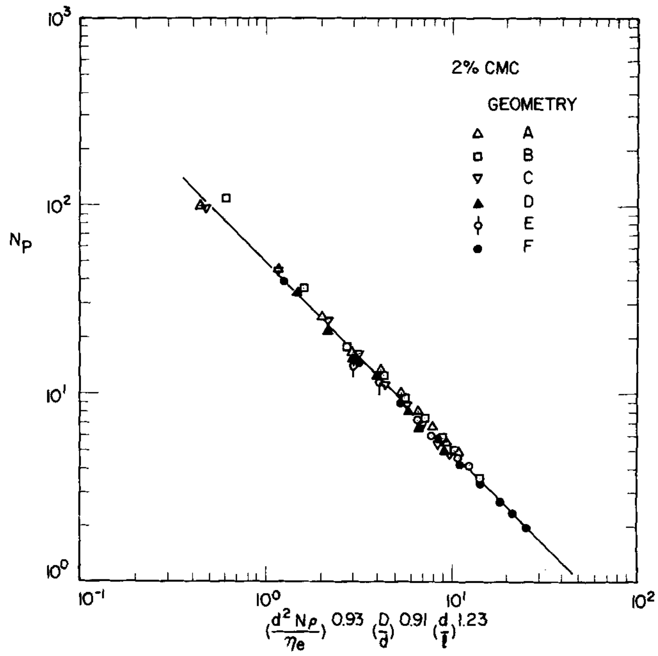

Fig. 2. Generalized correlation of the power number for the CMC solution in various geometries. Equation (13) is represented by the solid line.

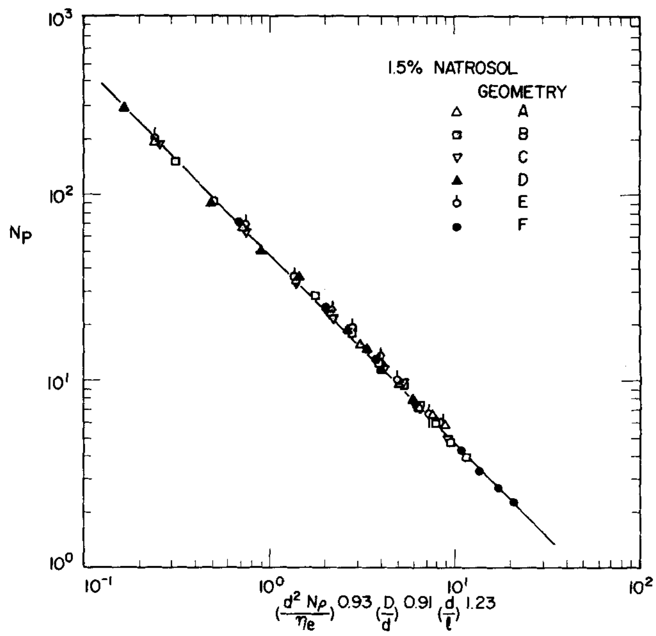

Fig. 3. Generalized correlation of the power number for the 1.5% Natrosol solution in various geometries. Equation (13) is represented by the solid line.

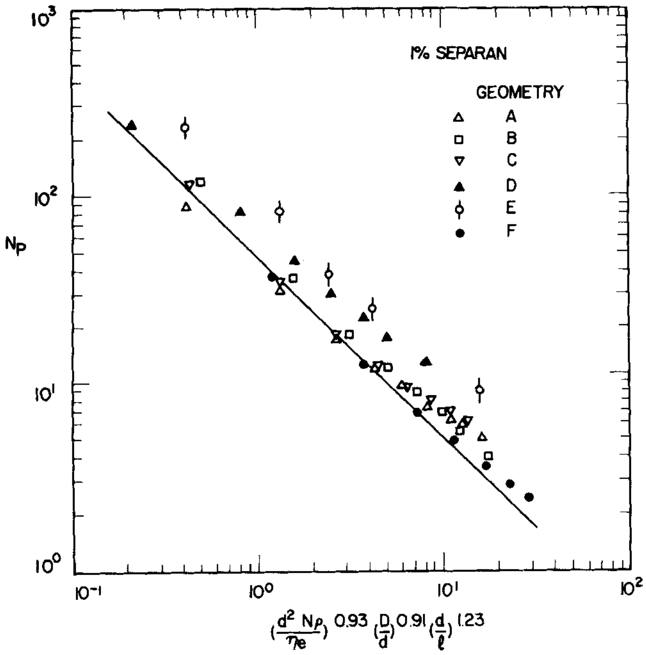

Fig. 4. Generalized correlation of the power number for the 1% Separan solution in various geometries. Equation (13), is represented by the solid line.

The effective rate of deformation is then given by

$$\dot{\gamma}_e = \phi_1 N = 4^{1/2S} \left[\left(\frac{d}{D}\right)^2 \left(\frac{l}{d}\right) \right] N \qquad (12)$$

With this expression for the effective rate of deformation, the effective viscosity is readily obtained from viscometric data or calculated with the aid of Equation (6) and the generalized Reynolds number defined by Equation (7).

Generalized Power Correlation

The correlation proposed by Patterson et al. (1979) can be generalized to take into account the effective viscosity of the viscoelastic fluids. The conventional Reynolds number in their Equation (21) is replaced by the generalized Reynolds number to obtain the generalized correlation for the power correlation for viscoelastic fluids:

$$N_p = \frac{P}{\rho N^3 d^5} = 24\, n_b \left[(Re_g)^{0.93} \left(\frac{D}{d}\right)^{0.91} \left(\frac{d}{l}\right)^{1.23} \right]^{-1} \qquad (13)$$

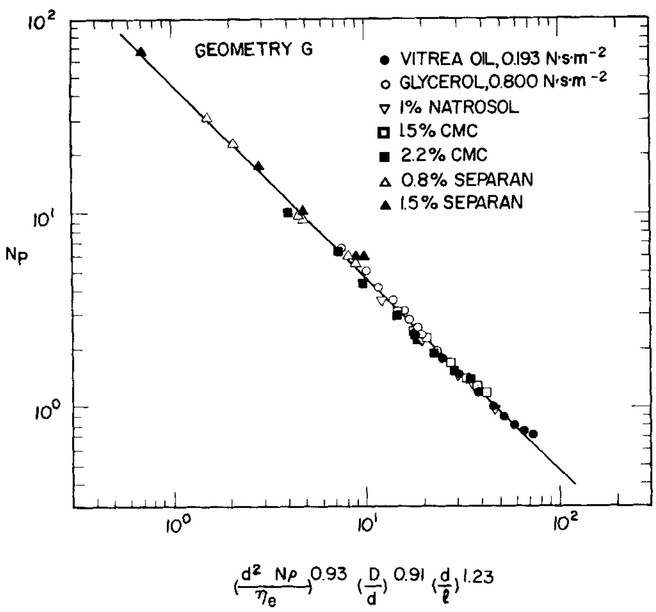

Fig. 5. Generalized correlation of the power number for various fluids in geometry G. Equation (13) is represented by the solid line.

TABLE 3. RELATIVE EFFICIENCY OF VARIOUS IMPELLERS

					(Efficiency)$_{rel}$	
Impeller*	n_b	D/d	l/d	w/d	2% CMC	1% Separan
I	2	1.11	4.48	0.097	0.0 28	0.0 22
II	2	1.11	3.00	0.097	0.0 51	0.0 38
III	2	1.11	4.12	0.195	0.0 59	0.0 30
IV	2	1.37	4.00	0.121	0.0 27	0.0 35
V	1	1.11	4.39	0.097	0.0 46	0.0 36

* For a more detailed description of the impellers, refer to Patterson et al. (1979), Table 1.

This correlation is preferable to the one that would be obtained from their alternative Equation (24) because it gives smaller deviations and has some theoretical foundations. The fit of the data, illustrated in Figures 2 and 3 for the CMC and Natrosol solutions, is remarkably good. The correlation for the 2% CMC and 1.5% Natrosol solutions have standard deviations of 6 and 4%, respectively. The 1% Separan (Figure 4) shows larger deviations, especially for geometries D and E. Excluding the results of geometries D and E, the standard deviation of the correlation of 1% Separan is 14%. This is not surprising, as the expression used for the effective rate of deformation is too simplistic for highly elastic fluids such as polyacrylamide (Separan) solutions. The elastic nature of this fluid plays an important role in its stress response. We note that Chavan and Ulbrecht (1973) using helical screw agitators (with and without draught tubes) did not observe any effect of fluid elasticity on power consumption.

In addition, two more Newtonian fluids and five other viscoelastic solutions were tested to judge the validity of Equation (13) for the impeller geometry G. The fit shown in Figure 5 is remarkable when we consider the extent of the rheological properties of the seven fluids used (see Table 1). The standard deviation was found to be 5%. The important geometrical variables of this correlation are the vessel diameter D, the agitator diameter d, and the blade length l. The important rheological parameters are η_0, the zero-shear viscosity; t_1, a fluid time constant; and S, a dimensionless parameter associated with the non-Newtonian behavior. When the rate of deformation is large enough, these three parameters reduce to the well-known power law parameters n and m:

$$1 - 2S = n \qquad (14)$$

$$\frac{\eta_0}{|t_1|^{2S}} = m \qquad (15)$$

Relative Efficiency

The relative efficiency of an impeller, as discussed by Patterson et al. (1979), is a useful measure for comparing different mixer geometries.

The higher the relative efficiency, the less total energy is required to achieve a certain degree of mixing in a given volume of fluid. In Table 3, the performance of five impellers in the 0.15 m vessel is given. This indicates that impeller III, which has wider blades (w is twice that of the others), is the most efficient impeller. For the Newtonian glycerol, its efficiency is 2.5 times of that of impeller I. However, its advantage vanishes as the fluid elasticity increases (for example, 1% Separan). For Newtonian or low elasticity fluids, a larger blade width results in a higher pumping capacity, but the torque is unchanged. For the highly elastic Separan solution, the pumping capacity of impeller III is not appreciably increased (Carreau et al., 1976). The relative efficiency of impeller II is rather good as it consumes low power, since the ratio l/d is rather small (high pitch of the blades) compared with the others. The one-blade impeller V has only a slightly lower pumping capacity or lower mixing effectiveness; however, since its power consumption is approximately half of that consumed by impeller I, its efficiency is almost doubled.

For elastic fluids, the mixing efficiency is not strongly dependent on the impeller geometry. However, it decreases rapidly as the fluid elasticity increases. The mixing efficiency in the 1% Separan solution is only 20 to 40% of that in glycerol.

Scale-up and Design

It has been remarked previously by Patterson et al. (1979) that valid scale-up is dependent on preserving both kinematic and dynamic similarity. In that paper, it was shown that for geometrically similar mixers, the criterion to be observed is that the power per unit volume in the two mixers be equal for equal degrees of homogeneity of the mix. This was based on the inverse relation between the power number and the Reynolds number and the fact that the number of impeller revolutions to achieve a given degree of homogeneity is constant. These conditions are satisfied for the non-Newtonian fluids (using Re_g), but serious deviations are observed for the highly viscoelastic fluids (see Figures 1 and 4).

The scale-up problem for viscoelastic fluids should thus be approached with caution. The rate of deformation in the vicinity of the blades increases with increasing scale, and shear degradation of the fluid is probable in very large mixers.

Effect of Mixer Geometry

Table 3 summarizes the effect of geometry on the mixing process. It is seen that the pseudoplastic fluids respond in a manner similar to the Newtonian fluids to changes in geometry, although the impeller efficiency is drastically reduced. The outstanding features are that efficiency is primarily influenced by blade width and that one-blade impellers could be very interesting from the point of view of cost.

Highly viscoelastic fluids are remarkable for their insensitivity to different geometries. Compared with the Newtonian fluids behavior, all the agitators are inefficient when mixing viscoelastic fluids, in spite of their reputed effectiveness in handling difficult liquids. This serves to underline the difficulty of achieving homogeneous mixtures of viscoelastic fluids.

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NOTATION

d = diameter of impeller, m
D = diameter of vessel, m
Fr = Froude number = N^2d/g, dimensionless
g = gravitational constant, ms^{-2}
k = proportionality constant, Equation (1), dimensionless
l = length of impeller blade, m
m = power law consistency index, $N \cdot s^n \cdot m^{-2}$
n = power law exponent, dimensionless
n_b = number of blades
N = agitator rotational speed, s^{-1}
N_p = power number $P/\rho N^3 d^5$, dimensionless
P = power, W
Re = Reynolds number $d^2N\rho/\mu$, dimensionless
Re_g = generalized Reynolds number, dimensionless
S = fluid rheological parameter, Equation (6), dimensionless
t_1 = fluid characteristic time, Equation (6), s

Greek Letters

$\dot{\gamma}_e$ = effective shear rate, Equation (1), s^{-1}
$\dot{\gamma}$ = shear rate, s^{-1}
η = viscosity, $N \cdot s \cdot m^{-2}$
η_e = effective viscosity, $N \cdot s \cdot m^{-2}$
η_0 = zero shear viscosity, $N \cdot s \cdot m^{-2}$
ϕ_1 = proportional factor, Equation (8), dimensionless
ϕ_2 = shift factor defined by Equation (10), dimensionless
ρ = fluid density, $kg \cdot m^3$
τ_{12} = shear stress, $N \cdot m^{-2}$
$\tau_{11} - \tau_{22}$ = primary normal stress difference, $N \cdot m^{-2}$

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